Head Office: 2nd Floor, Grand Plaza, Fraser Road, Dak Bunglow, Patna - 01

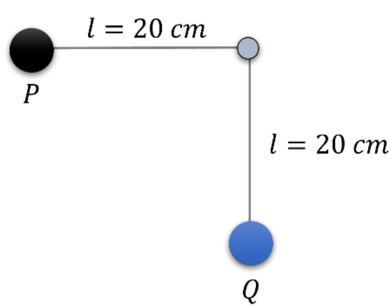
JEE Main 2023 (Memory based)

30 January 2023 - Shift 1

Answer & Solutions

PHYSICS

1. Bob P is released from the position of rest at the moment shown. If it collides elastically with an identical bob Q hanging freely then velocity of Q just after collision is $(g = 10\ m/s^2)$



Answer (C)

Solution:

Velocity of P just before collision is $\sqrt{2gl} = 2\ m/s$
As collision is elastic and the mass of P and Q is equal therefore just after collision,
velocity of P is 0 and that of Q is $2\ m/s$.

2. Choose the option showing the correct relation between Poisson's ratio(σ), Bulk modulus(B) and Modulus of rigidity(G).

A. $\sigma = \frac{3B-2G}{2G+6B}$

B. $\sigma = \frac{6B+2G}{3B-2G}$

C. $\sigma = \frac{9BG}{3B+G}$

D. $B = \frac{3\sigma-3G}{6\sigma+2G}$

Answer (A)

Solution:

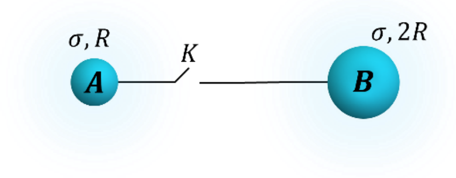
$E = 2G(1+\sigma) \ldots\ldots (1)$
$E = 3B(1-2\sigma) \ldots\ldots (2)$

$$1 = \frac{2G}{3B}\left(\frac{1+\sigma}{1-2\sigma}\right)$$

$$3B - 6B\sigma = 2G + 2G\sigma$$
$$3B - 2G = \sigma(2G + 6B)$$
$$\sigma = \frac{3B - 2G}{2G + 6B}$$

3. Two conducting solid spheres A and B are placed at a very large distance with charge densities and radii as shown. When the key K is closed, find the ratio of final charge densities.

A. $4:1$
B. $1:2$
C. $2:1$
D. $1:4$



Answer (C)

Solution:

Final potential is same:

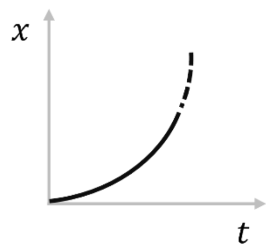
$$\Rightarrow \frac{1}{4\pi\epsilon_0} \times \frac{Q_1}{R} = \frac{1}{4\pi\epsilon_0} \times \frac{Q_2}{2R} \ldots\ldots..(1)$$

Also, $\quad Q_1 + Q_2 = \sigma.4\pi R^2 + \sigma.4\pi(2R)^2 \ldots\ldots(2)$

From (1) and (2)

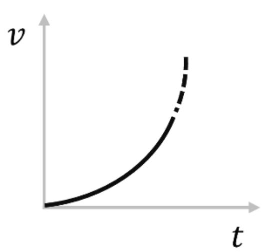
$$\frac{\sigma_1}{\sigma_2} = 2$$

4. Position-time graph for a particle is parabolic and is as shown:

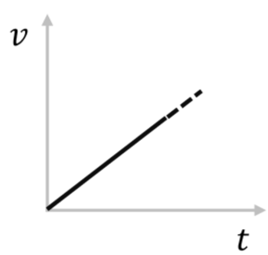


Choose the corresponding v-t graph.

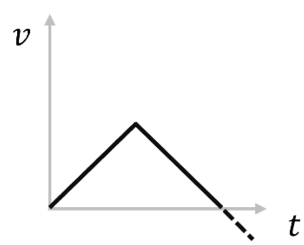
A.



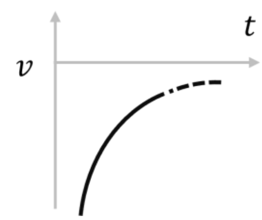
B.



C.



D.



Answer (B)

Solution:

Since, $x \propto t^2$

$$\Rightarrow v = \frac{dx}{dt} \propto t$$

So, it will be a linear plot and option (B) is correct.

5. Electromagnetic wave beam of power $20\ mW$ is incident on a perfectly absorbing body for $300\ ns$. The total momentum transferred by the beam to the body is equal to

A. $2\times10^{-17}\ Ns$
B. $1\times10^{-17}\ Ns$
C. $3\times10^{-17}\ Ns$
D. $5\times10^{-17}\ Ns$

Answer (A)

Solution:

Total energy incident $= Pt$

So,

Total initial momentum $= Pt/c$

Total final momentum $= 0$

Total momentum transferred:

$$\frac{Pt}{c}=\frac{20\times10^{-3}\times300\times10^{-9}}{3\times10^{8}}=2\times10^{-17}\ Ns$$

6. The velocity of an electron in the seventh orbit of hydrogen like atom is $3.6\times10^{6}\ m/s$. Find the velocity of the electron in the $3rd$ orbit.

A. $4.2\times10^{6}\ m/s$
B. $8.4\times10^{6}\ m/s$
C. $2.1\times10^{6}\ m/s$
D. $3.6\times10^{6}\ m/s$

Answer (B)

Solution:

For hydrogen like atom,

$$v\propto\frac{1}{n}$$

$$\frac{v_1}{v_2}=\frac{n_2}{n_1}$$

$$\frac{3.6\times10^{6}}{v_2}=\frac{3}{7}$$

$$v_2=8.4\times10^{6}\ m/s$$

7. Electric field in a region is given by $\vec{E}=\frac{a}{x^2}\ \hat{\imath}+\frac{b}{y^3}\hat{\jmath}$, where $x\ \&\ y$ are coordinates. Find SI units of $a\ \&\ b$.

A. $a-Nm^2C^{-1}, b-Nm^3C^{-1}$
B. $a-Nm^3C^{-1},, b-Nm^2C^{-1}$
C. $a-Nm\ C^{-1},, b-Nm^2C^{-1}$
D. $a-Nm^2C^{-1},, b-Nm^2C^{-1}$

Answer (A)

Solution:

Unit of Electric Field: $E\Rightarrow N/C$

x^2-m^2

y^3-m^3

$a-Nm^2C^{-1},, b-Nm^3C^{-1}$

8. Coil A of radius $10\ cm$ has N_A number of turns and I_A current is flowing through it. Coil B of radius $20\ cm$ has N_B number of turns and I_B current is flowing through it. If magnetic dipole moment of both the coils is same then

A. $I_A N_A = 4 I_B N_B$
B. $I_A N_A = \frac{1}{4} I_B N_B$
C. $I_A N_A = 2 I_B N_B$
D. $I_A N_A = \frac{1}{2} I_B N_B$

Answer (A)

Solution:

Magnetic dipole moment, $\mu = NiA = Ni\pi R^2$

So,

$$\frac{\mu_A}{\mu_B} = \frac{N_A i_A R_A^2}{N_B i_B R_B^2} = 1$$

$$\frac{N_A i_A 10^2}{N_B i_B 20^2} = 1$$

$$N_A i_A = 4 N_B i_B$$

9. An ideal gas undergoes a thermodynamic process following the relation $PT^2 = constant$. Assuming symbols have their usual meaning then volume expansion coefficient of the gas is equal to:

A. $2/T$
B. $3/T$
C. $1/2T$
D. $1/T$

Answer (C)

Solution:

Volume expansion coefficient= $V \times \left(\frac{dV}{dT}\right)$

So,

$PT^2 = constant$

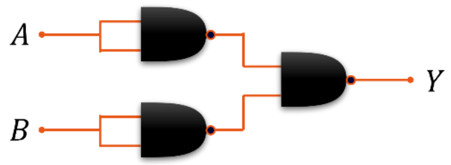
Or,

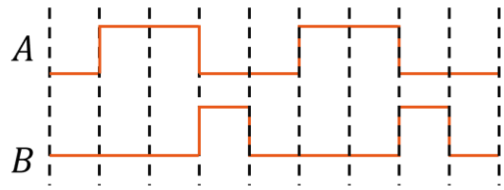
$$\frac{T^3}{V} = constant$$

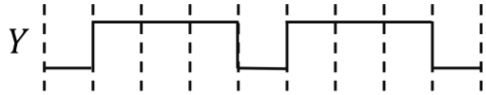
$$\frac{dV}{dT} = C \times 3T^2$$

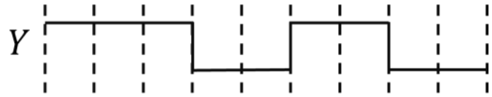
$$\frac{1}{V} \times \frac{dV}{dT} = \frac{3T^2}{T^3} = \frac{3}{T}$$

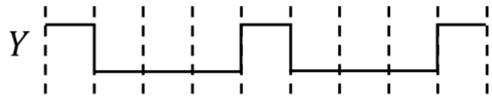
10. Consider a combination of gates as shown:





A. 

B. 

C. 

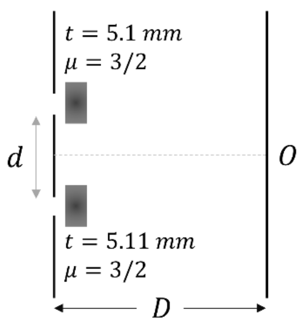
D. 

Answer (A)

Solution:

$Y = (A'B')' = A + B$
⇒OR gate

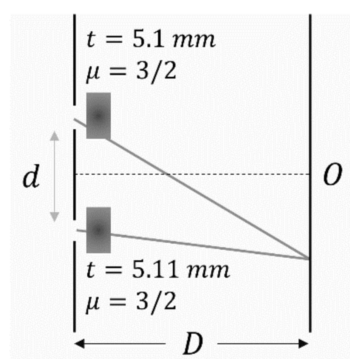
11. For the given $YDSE$ setup, find the number of fringes by which the central maxima gets shifted from point O.
(Given $d = 1\ mm,\ D = 1\ m,\ \lambda = 5000\dot{A}$)



A. 10
B. 15
C. 8
D. 12

Answer (A)

Solution:



At central position, path difference is,
$\Delta x = (\mu - 1)(t_1 - t_2)$
$\Delta x = \left(\frac{3}{2} - 1\right)(5.11 - 5.1)\ mm = \frac{1}{2} \times 0.01 = 0.005\ mm$

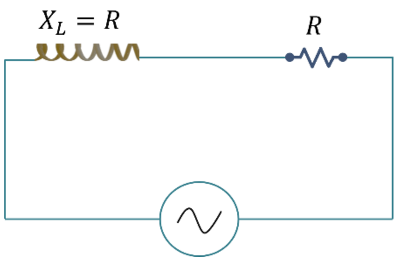
Number of fringes shifted= $\Delta x/\lambda$
$\frac{\Delta x}{\lambda} = \frac{5 \times 10^{-6}}{5 \times 10^{-7}} = 10$

12. If an insulator with inductive reactance $X_L = R$ is connected in series with resistance R across an $A.C.$ Voltage, power factor comes out to be P_1. Now, if a capacitor with capacitive reactance $X_C = R$ is also connected in series with the inductor and resistor in the same circuit, power factor becomes P_2. Find P_1/P_2.

A. $\sqrt{2}:1$
B. $1:\sqrt{2}$
C. $1:1$
D. $1:2$

Answer (B)

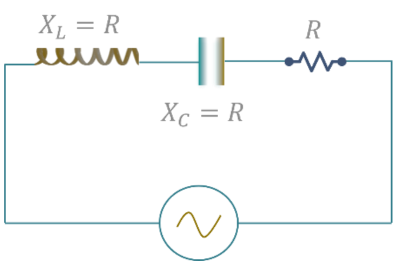
Solution:



Impedance of circuit:

$Z = \sqrt{R^2 + R^2} = \sqrt{2}R$

$P_1 = \cos\phi = \text{Power Factor} = \frac{R}{Z} = \frac{1}{\sqrt{2}}$



When capacitor is also connected in series:

The LCR circuit is in resonance stage

So,

$Z = \sqrt{R^2 + (X_L - X_C)^2} = R$

$P_2 = \cos\phi = Power\ Factor = \frac{R}{Z} = \frac{R}{R} = 1$

So,

$\frac{P_1}{P_2} = \frac{\left(\frac{1}{\sqrt{2}}\right)}{1} = \frac{1}{\sqrt{2}}$

13. For a system undergoing isothermal process, heat energy is supplied to the system. Choose the option showing correct statements.

1) Internal energy will increase.
2) Internal energy will decrease.
3) Work done by system is positive.
4) Work done by system is negative.
5) Internal energy remains constant.

A. (1), (3), (5)
B. (2), (4)
C. (3), (5)
D. (1), (4), (5)

Answer (C)

Solution:

From First Law of thermodynamics, $\Delta Q = \Delta U + \Delta W$
For Isothermal process, $\Delta T = 0 \Rightarrow U = Constant \Rightarrow \Delta U = 0$
So,
$\Delta Q = \Delta W$
As heat is supplied, ΔW is positive. Hence, work is done by the system.

14. The heat passing through the cross-section of a conductor, varies with time 't' as $Q(t) = \alpha t - \beta t^2 + \gamma t^3 (\alpha, \beta$ and γ are positive constants). The minimum heat current through the conductor is

A. $\alpha - \frac{\beta^2}{2\gamma}$
B. $\alpha - \frac{\beta^2}{3\gamma}$
C. $\alpha - \frac{\beta^2}{\gamma}$
D. $\alpha - \frac{3\beta^2}{\gamma}$

Answer (B)

Solution:

Heat through cross section of rod, $Q = \alpha t - \beta t^2 + \gamma t^3$
Heat current, $i = \frac{dQ}{dt} = \alpha - 2\beta t + 3\gamma t^2$

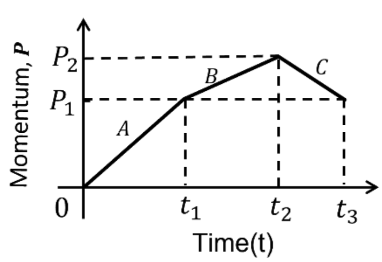
For heat current to be minimum, $\frac{di}{dt} = 0$
$\Rightarrow -2\beta + 6\gamma t = 0$
$\Rightarrow t = \dfrac{\beta}{3\gamma}$

So minimum heat current, $i_{min} = \alpha - 2\beta \times \frac{\beta}{3\gamma} + 3\gamma \left(\frac{\beta}{3\gamma}\right)^2$

$$= \alpha - \frac{\beta^2}{3\gamma}$$

15. Momentum-time graph of an object moving along a straight line is as shown in figure. If $(P_2 - P_1) < P_1$ and $(t_2 - t_1) = t_1 < (t_3 - t_2)$ then at which points among A, B and C the magnitude of force experienced by the object is maximum and minimum respectively

A. A, B
B. A, C
C. B, C
D. B, A



Answer (B)

Solution:

$$F_A = \frac{P_1 - 0}{t_1 - 0}$$
$$F_B = \frac{P_2 - P_1}{t_2 - t_1}$$

$$|F_C| = \frac{P_2 - P_1}{t_3 - t_2}$$

As given, $(P_2 - P_1) < P_1$ and $(t_2 - t_1) = t_1$ So, $F_B < F_A$

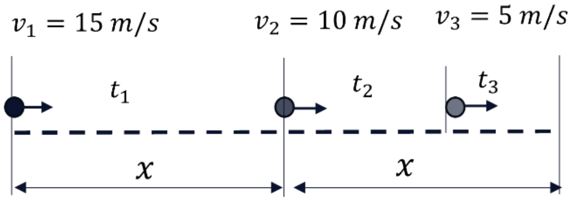
As given, $(t_2 - t_1) < (t_3 - t_2)$ So, $F_B > |F_C|$

16. A particle moving in unidirectional motion travels half of the total distance with a constant speed of $15\ m/s$. Now first half of the remaining journey time, it travels at $10\ m/s$ and second half of the remaining journey time, it travels at $5\ m/s$. Average speed of the particle is:

A. $12\ m/s$
B. $10\ m/s$
C. $7\ m/s$
D. $9\ m/s$

Answer (B)

Solution:



$$v_{av} = \frac{2x}{t_1 + t_2 + t_3}$$

As $t_2 = t_3 = t$

So, $x = t(v_2 + v_3) \Rightarrow t = x/(v_2 + v_3)$

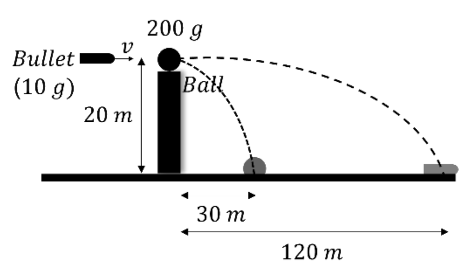
$$v_{av} = \frac{2x}{\frac{x}{15} + 2t}$$

$$v_{av} = \frac{2x}{\frac{x}{15} + \frac{2x}{10+5}}$$

$$v_{av} = 10\ m/s$$

17. A bullet strikes a stationary ball kept at a height as shown. After collision, range of bullet is $120\ m$ and that of ball is $30\ m$. Find initial speed of bullet. Collision is along horizontal direction.$(Take\ g = 10\ m/s^2)$

A. $150\ m/s$
B. $90\ m/s$
C. $240\ m/s$
D. $360\ m/s$



Answer (D)

Solution:

Applying conservation of momentum,
$m_1 v + m_2(0) = m_1 v_1' + m_2 v_2'$

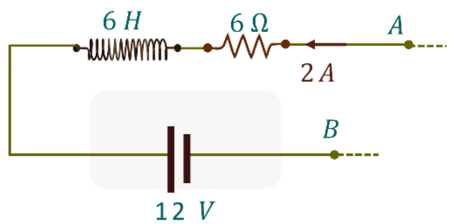
$$\Delta t = \sqrt{\frac{2h}{g}} = 2\ s$$

$$v_1' = \frac{120}{2} = 60\ m/s$$

$$v_2' = \frac{30}{2} = 15\, m/s$$
$$v = 360\, m/s$$

18. In a part of circuit as shown, find $V_A - V_B$ in $Volts$. It is given that current is decreasing at a rate of $1\, Ampere/s$

Answer (18)



Solution:

$$V_A - iR - \frac{Ldi}{dt} - 12 = V_b$$
$$V_A - 2 \times 6 - 6(-1) \; - 12 = V_B$$
$$V_A - V_B = 18$$

19. A particle undergoing SHM, follows the position-time equation given as $x = A\sin(\omega t + \pi/3)$. If the SHM motion has a time period of T, then velocity will be maximum at time $t = T/\beta$ for first time after $t = 0$. value of β is equal to ______.

Answer (3)

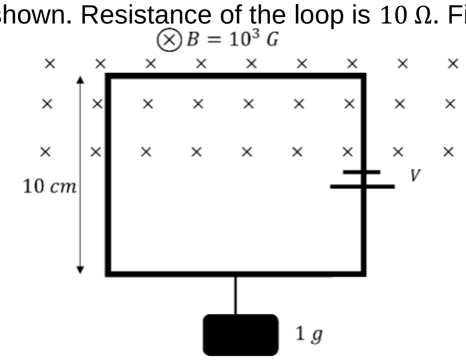
Solution:

$$x = A\, Sin\, (\omega t + \frac{\pi}{3})$$
$$v = A\omega Cos\left(\omega t + \frac{\pi}{3}\right)$$

For maximum value of v,

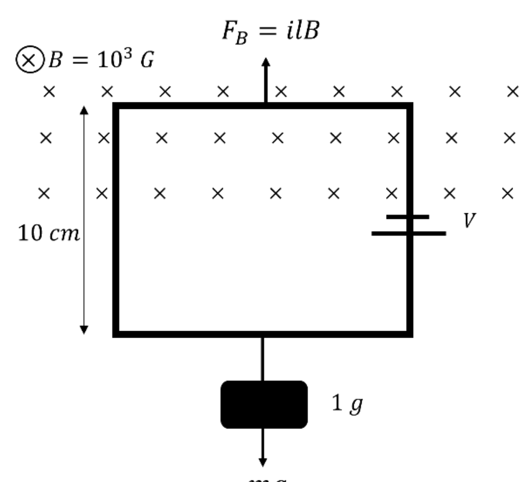
$$Cos\, \left(\omega t + \frac{\pi}{3}\right) = \pm 1$$
$$\Rightarrow \omega t + \frac{\pi}{3} = \pi$$
$$\Rightarrow \omega t = \frac{2\pi}{3}$$
$$\Rightarrow t = \frac{T}{3}$$

20. A block of mass $1\, kg$ is in equilibrium with the help of current carrying square loop which is partially laying in constant magnetic field (B) as shown. Resistance of the loop is $10\, \Omega$. Find the voltage V of the battery in the loop.



Answer (10)

Solution:



In equilibrium, $ilB = mg$

$$\Rightarrow i = \frac{mg}{lB} = \frac{1 \times 10^{-3}\, kg \times 10\frac{m}{s^2}}{0.1\, m \times 0.1\, T} = 1\, A$$

As resistance of loop =1 Ω

$$V = iR = 10\, V$$

21. Initial volume of $1\ mole$ of a $monoatomic$ gas is $2\ litres$. It is expanded isothermally to a volume of $6\ litres$. Change in internal energy is xR. Find x.

Answer (0)

Solution:

As internal energy is a function of temperature only.
$\Delta U = nC_v\Delta T$
So, $\Delta U = 0$

22. An object is placed at a distance of $40\ cm$ from the pole of a converging mirror. The image is formed at a distance of $120\ cm$ from the mirror on the same side. The focal length is measured with a scale where each $1\ cm$ has 20 equal divisions. If the fractional error in the measurement of focal length is $1/10K$, then find K.

Answer (60)

Solution:

$u = -40\ cm\ \ v = -120\ cm$

Mirror formula,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow -\frac{1}{120} - \frac{1}{40} = \frac{1}{f}$$

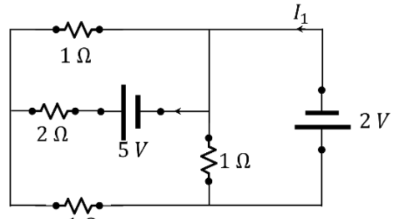
$$f = -30\ cm$$

$$Least\ count\ of\ scale = \frac{1}{20}\ cm$$

$$Fractional\ error\ = \frac{\left(\frac{1}{20}\right)}{30} = \frac{1}{600}$$

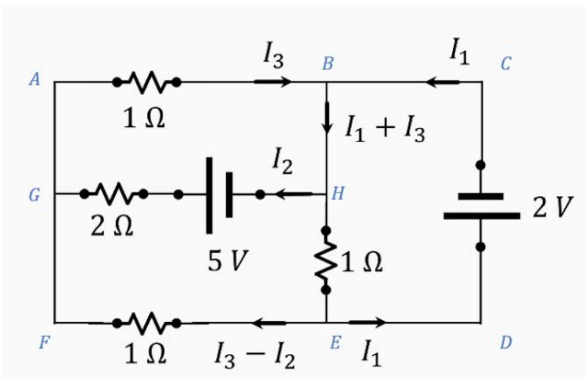
So, $K = 60$

23. In the circuit shown below, the value of $current\ I_1$(in $amperes$) is equal to $-\frac{y}{5}\ Amp$. The value of y is equal to:



Answer (11)

Solution:



Using Kirchhoff's voltage law,

For loop $BCDEHB$, $(I_1 + I_3 - I_2) \times 1 = -2 \ldots\ldots (1)$

For loop $ABHGA$, $(I_3 + 2I_2) = 5 \ldots\ldots\ldots\ldots\ldots\ldots (2)$

For loop $GHEFG$, $2I_2 - (I_3 - I_2) - (I_1 + I_3 - I_2) = 5 \ldots\ldots (3)$

Solving (1), (2) and (3)

$$I_1 = -\frac{11}{5}$$

$$y = 11$$